A branch-and-Benders-cut approach for the r-allocation p-hub maximal covering problem with binary and partial coverage criteria

Patrick Doglio

Pós-Graduaccão em Engenharia de Produção, Universidade Federal Fluminense, Brazil

Marcos Roboredo

Departamento de Engenharia de Produção, Universidade Federal Fluminense, Brazil

Artur Pessoa

Departamento de Engenharia de Produção, Universidade Federal Fluminense, Brazil

1. Introduction

Tables 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17 and 18 present computational results for three approaches: the best exact literature approach, the best exact literature approach with the proposed cuts being added on demand and the proposed BBC algorithm. The group of instances (CAB or AP) and the type of allocation (r or single) are indicated in the caption of each table. The following information is presented in each table: the number of network nodes (column n); the number of hubs to be located (column p); the maximum number of located hubs that a non-hub point can be allocated (column r); the value of the constant α (column α); the value of the constant β (column β); the optimal value for the objective function (column *opt*); the percentage root node gap (column $Gap_0(s)$); the total CPU time in seconds spent by the root node (column $T_0(s)$); the total CPU time in seconds (column $T(s)$); We set a time limit of 7200s for each experiment.

Email addresses: `patrickdoglio@id.uff.br` (Patrick Doglio), `mcroboredo@id.uff.br` (Marcos Roboredo), `artur@producao.uff.br` (Artur Pessoa)

Table 1: Comparison with the literature for CAB instances with r-allocation and binary coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	3	2	0.2	1913	0.9658	3.5	2.4	2.4	24.6	2.1	4.1	1.8	43.4	1.8	8.7
25	3	2	0.4	2099	0.9624	3.6	2.2	1.3	13.0	0.7	4.0	0.5	27.5	0.4	7.0
25	3	2	0.6	2336	0.9317	4.7	1.6	1.7	10.6	1.9	2.6	1.2	23.7	1.4	7.2
25	3	2	0.8	2552	0.9008	1.5	1.4	0.2	7.6	0.2	2.6	0.0	10.2	0.0	4.3
25	4	2	0.2	1617	0.9571	4.5	2.3	2.8	22.0	2.7	5.2	2.2	39.9	2.3	12.3
25	4	2	0.4	1881	0.9501	4.9	1.6	1.7	16.3	2.1	4.1	1.4	34.2	1.3	6.7
25	4	2	0.6	2184	0.9362	3.6	1.4	0.7	10.6	0.6	2.7	0.1	28.8	0.4	5.8
25	4	2	0.8	2457	0.8960	1.5	1.3	0.5	11.2	0.6	2.6	0.3	27.2	0.5	5.3
25	4	3	0.2	1617	0.9571	4.5	2.0	3.1	14.5	2.7	4.0	2.3	33.5	2.4	9.5
25	4	3	0.4	1881	0.9501	4.9	1.7	2.0	10.2	2.1	3.0	1.3	30.7	1.3	6.0
25	4	3	0.6	2184	0.9363	3.5	1.4	0.6	12.3	0.5	2.6	0.3	21.0	0.4	4.6
25	4	3	0.8	2457	0.8961	1.5	1.3	0.6	8.6	0.6	2.6	0.4	19.0	0.5	5.1
25	5	2	0.2	1346	0.9270	7.7	1.7	3.6	16.0	3.6	2.8	2.6	26.8	2.7	9.1
25	5	2	0.4	1597	0.9145	6.6	1.3	2.5	7.5	1.6	3.0	1.0	23.1	1.2	7.9
25	5	2	0.6	2002	0.9012	4.0	1.3	1.5	8.9	1.4	3.1	0.8	23.2	1.0	5.9
25	5	2	0.8	2307	0.8903	0.9	1.1	0.0	6.2	0.3	2.6	0.0	7.2	0.0	4.2
25	5	3	0.2	1346	0.9270	7.7	1.7	3.7	14.8	3.6	3.7	2.9	22.1	2.9	13.6
25	5	3	0.4	1597	0.9184	6.2	1.2	2.0	8.3	1.2	2.9	0.8	14.5	1.0	5.7
25	5	3	0.6	2002	0.9019	3.9	1.3	1.4	7.1	1.3	2.6	0.7	16.6	1.0	6.5
25	5	3	0.8	2307	0.8905	0.9	1.2	0.2	6.2	0.4	2.2	0.0	13.4	0.2	4.0
25	5	4	0.2	1346	0.9270	7.7	1.5	3.5	14.8	3.6	3.0	2.9	27.9	2.8	14.4
25	5	4	0.4	1597	0.9184	6.2	1.2	2.1	6.6	1.2	2.5	0.9	15.9	0.9	5.9
25	5	4	0.6	2002	0.9019	3.9	1.2	1.1	8.6	1.3	2.5	0.9	16.8	0.8	5.5
25	5	4	0.8	2307	0.8905	0.9	1.1	0.0	5.2	0.4	2.0	0.0	8.9	0.2	3.9
Avg.						4.1	1.5	1.6	11.3	1.5	3.0	1.1	23.1	1.1	7.0

Table 2: Comparison with the literature for AP25 instances with r-allocation and binary coverage criterion.

										Algorithm (1)					
						State-of-art Lit. Formulation						Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	3	2	0.2	40780.89	0.9623	3.9	3.8	3.2	30.8	2.6	4.0	1.8	60.8	2.0	10.9
25	3	2	0.4	41390.96	0.9467	5.2	2.8	3.2	27.4	2.6	2.9	1.9	45.7	2.3	9.9
25	3	2	0.6	43001.19	0.9372	4.6	1.9	2.2	18.1	2.0	2.7	1.5	26.2	1.7	6.5
25	3	2	0.8	48589.33	0.9686	1.2	1.8	0.5	9.2	0.6	2.9	0.3	17.8	0.4	5.5
25	4	2	0.2	33019.26	0.9314	7.3	2.4	5.2	21.1	4.0	3.5	3.4	32.1	3.3	12.7
25	4	2	0.4	36553.23	0.9546	4.0	1.7	1.9	14.5	1.4	4.1	0.8	23.9	1.0	7.7
25	4	2	0.6	38746.77	0.9297	4.2	1.5	1.8	15.4	2.0	2.6	1.5	33.3	1.8	9.1
25	4	2	0.8	48589.33	0.9806	0.8	1.7	0.5	14.0	0.5	2.5	0.5	31.4	0.5	6.3
25	4	3	0.2	33019.26	0.9314	7.3	2.4	5.0	22.9	4.0	2.8	3.4	45.3	3.5	12.1
25	4	3	0.4	36553.23	0.9571	3.7	1.6	1.5	15.0	1.1	2.6	0.5	20.0	0.7	6.0
25	4	3	0.6	38746.77	0.9378	3.3	1.4	0.8	12.5	1.1	2.4	0.8	18.9	0.9	4.9
25	4	3	0.8	48589.33	0.9859	0.4	1.4	0.1	6.9	0.1	2.6	0.0	7.5	0.0	3.9
25	5	2	0.2	30218.49	0.9534	4.9	2.3	3.1	13.7	2.2	4.5	1.3	30.5	1.5	8.3
25	5	2	0.4	32335.62	0.9347	5.4	1.4	1.9	11.8	2.2	2.6	1.3	24.8	1.7	6.8
25	5	2	0.6	36682.65	0.9270	4.2	1.3	2.6	10.6	2.5	2.4	2.0	30.8	2.3	11.2
25	5	2	0.8	48589.33	0.9901	0.3	1.4	0.1	12.2	0.2	2.4	0.1	21.8	0.1	4.6
25	5	3	0.2	30218.49	0.9534	4.9	2.0	3.1	12.8	2.2	3.1	1.2	31.9	1.5	6.9
25	5	3	0.4	32335.62	0.9392	4.9	1.5	1.6	12.5	1.7	2.5	1.2	19.6	1.2	10.5
25	5	3	0.6	36682.65	0.9367	3.1	1.3	1.4	8.7	1.5	2.4	1.1	20.3	1.2	5.5
25	5	3	0.8	48589.33	0.9901	0.4	1.4	0.2	7.5	0.2	2.2	0.2	15.0	0.2	4.9
25	5	4	0.2	30218.49	0.9534	4.9	1.8	2.9	14.1	2.2	2.8	1.2	25.1	1.4	7.5
25	5	4	0.4	32335.62	0.9392	4.9	1.5	1.5	11.8	1.7	2.5	1.0	22.2	1.1	6.0
25	5	4	0.6	36682.65	0.9367	3.1	1.3	1.4	11.2	1.5	2.3	1.0	23.3	1.3	6.3
25	5	4	0.8	48589.33	0.9901	0.4	1.3	0.2	5.5	0.2	2.2	0.1	16.8	0.1	4.4
Avg.						3.6	1.8	1.9	14.2	1.7	2.8	1.2	26.9	1.3	7.4

Table 3: Comparison with the literature for AP50 instances with r-allocation and binary coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
50	3	2	0.2	45698.26	0.9708	3.0	940.3	2.8	7200.0	2.0	82.1	1.9	7200.0	1.9	382.1
50	3	2	0.4	50870.54	0.9840	1.5	1020.3	1.3	7200.0	0.7	59.7	0.6	7200.0	0.7	164.3
50	3	2	0.6	54337.55	0.9836	1.2	444.8	1.0	7200.0	0.6	53.7	0.5	7200.0	0.5	105.6
50	3	2	0.8	56061.85	0.9786	0.9	447.3	0.8	5047.7	0.5	36.0	0.5	7200.0	0.5	72.0
50	4	2	0.2	39390.22	0.9793	2.0	551.7	1.7	7200.0	0.8	479.0	0.8	7200.0	0.8	667.0
50	4	2	0.4	45575.81	0.9850	1.3	585.5	1.1	7200.0	0.6	54.8	0.5	7200.0	0.6	93.4
50	4	2	0.6	48412.31	0.9762	1.6	438.7	1.3	7200.0	1.0	85.4	0.9	7200.0	0.9	174.8
50	4	2	0.8	54489.92	0.9799	0.9	407.8	0.8	7200.0	0.7	68.9	0.7	7200.0	0.6	192.8
50	4	3	0.2	39390.22	0.9804	1.9	517.1	1.6	7200.0	0.7	86.2	0.6	7200.0	0.7	309.2
50	4	3	0.4	45575.81	0.9855	1.3	435.5	1.1	7200.0	0.6	39.3	0.5	7200.0	0.5	236.7
50	4	3	0.6	48412.31	0.9779	1.4	460.6	1.2	7200.0	0.8	57.4	0.8	7200.0	0.8	95.0
50	4	3	0.8	54489.92	0.9831	0.6	388.1	0.5	5841.0	0.3	19.6	0.3	7200.0	0.3	37.0
50	5	2	0.2	32648.59	0.9478	5.1	164.3	4.2	7200.0	3.0	129.5	2.9	7200.0	2.9	715.8
50	5	2	0.4	39975.05	0.9684	2.8	175.7	2.4	7200.0	1.8	62.6	1.8	7200.0	1.8	429.8
50	5	2	0.6	45784.24	0.9753	1.7	325.5	1.4	7200.0	1.2	76.1	1.2	7200.0	1.2	360.8
50	5	2	0.8	54088.39	0.9845	0.7	411.1	0.6	7200.0	0.5	43.8	0.5	7200.0	0.5	217.0
50	5	3	0.2	32648.59	0.9483	5.1	217.5	4.2	7200.0	2.9	39.4	2.8	7200.0	2.8	1440.2
50	5	3	0.4	39975.05	0.9707	2.6	335.8	2.1	7200.0	1.6	35.3	1.5	7200.0	1.5	222.8
50	5	3	0.6	45784.24	0.9771	1.5	461.3	1.3	7200.0	1.0	36.2	1.0	7200.0	1.0	178.3
50	5	3	0.8	54088.39	0.9868	0.5	421.0	0.4	7200.0	0.3	33.0	0.3	7200.0	0.3	109.5
50	5	4	0.2	32648.59	0.9483	5.1	178.2	4.1	7200.0	2.9	32.8	2.8	7200.0	2.8	357.6
50	5	4	0.4	39975.05	0.9707	2.6	280.1	2.1	7200.0	1.6	35.9	1.5	7200.0	1.5	158.8
50	5	4	0.6	45784.24	0.9771	1.5	295.4	1.3	7200.0	1.0	25.9	1.0	7200.0	1.0	96.5
50	5	4	0.8	54088.39	0.9875	0.4	366.2	0.3	7200.0	0.2	19.8	0.2	7200.0	0.2	37.2
Avg.						2.0	427.9	1.6	7053.7	1.1	70.5	1.1	7200.0	1.1	285.6

Table 4: Comparison with the literature for CAB instances with r-allocation and partial coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	3	2	0.2	1913	0.9826	1.8	14.2	1.4	80.1	1.0	10.0	0.9	97.4	0.8	28.8
25	3	2	0.4	2099	0.9723	2.8	12.8	1.5	50.5	0.8	12.8	0.7	82.0	0.7	18.4
25	3	2	0.6	2336	0.9575	3.6	13.0	0.8	51.6	0.6	10.3	0.4	53.7	0.5	16.2
25	3	2	0.8	2552	0.9366	2.4	9.9	0.4	29.8	0.2	8.9	0.2	50.3	0.2	14.1
25	4	2	0.2	1617	0.9714	2.9	15.0	2.1	65.4	1.9	10.1	1.6	92.4	1.8	29.7
25	4	2	0.4	1881	0.9690	3.1	9.0	1.2	39.9	1.0	11.4	0.7	77.3	0.8	28.0
25	4	2	0.6	2184	0.9599	3.0	7.9	0.6	34.9	0.2	9.8	0.1	47.9	0.1	15.3
25	4	2	0.8	2457	0.9358	1.8	9.7	0.0	20.3	0.0	7.8	0.0	14.4	0.0	9.6
25	4	3	0.2	1617	0.9714	2.9	9.8	2.2	60.8	1.9	9.6	1.6	77.3	1.8	28.7
25	4	3	0.4	1881	0.9690	3.1	8.8	1.3	37.3	1.0	10.1	0.7	83.5	0.9	18.6
25	4	3	0.6	2184	0.9612	2.9	8.2	0.4	35.6	0.1	10.2	0.0	29.4	0.0	13.4
25	4	3	0.8	2457	0.9358	1.8	7.0	0.0	23.9	0.0	7.7	0.0	15.6	0.0	9.3
25	5	2	0.2	1346	0.9491	5.3	5.9	3.1	40.4	3.1	11.3	2.6	97.1	2.7	74.5
25	5	2	0.4	1597	0.9504	4.2	5.2	0.7	28.1	0.5	11.7	0.3	44.5	0.4	24.1
25	5	2	0.6	2002	0.9456	3.0	6.6	0.4	30.4	0.3	10.6	0.1	37.7	0.2	20.5
25	5	2	0.8	2307	0.9140	1.3	7.4	0.2	32.3	0.2	9.0	0.2	54.8	0.2	19.6
25	5	3	0.2	1346	0.9491	5.3	5.1	3.3	40.2	3.1	10.6	2.4	98.1	2.6	46.3
25	5	3	0.4	1597	0.9512	4.1	5.6	0.7	27.1	0.4	10.8	0.2	35.2	0.3	16.0
25	5	3	0.6	2002	0.9461	2.9	5.4	0.4	26.9	0.2	9.5	0.1	33.1	0.1	13.0
25	5	3	0.8	2307	0.9151	1.2	6.2	0.1	25.5	0.2	6.9	0.1	33.4	0.2	12.1
25	5	4	0.2	1346	0.9491	5.3	5.4	3.1	40.1	3.1	9.9	2.6	96.9	2.7	46.9
25	5	4	0.4	1597	0.9512	4.1	4.2	0.8	22.0	0.4	8.8	0.2	36.8	0.2	15.9
25	5	4	0.6	2002	0.9461	2.9	6.1	0.4	25.6	0.2	11.0	0.1	30.3	0.1	14.7
25	5	4	0.8	2307	0.9151	1.2	5.7	0.2	23.2	0.2	5.5	0.1	30.1	0.1	15.9
Avg.						3.0	8.1	1.1	37.2	0.9	9.8	0.7	56.2	0.7	22.9

Table 5: Comparison with the literature for AP25 instances with r-allocation and partial coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	3	2	0.2	40780.89	0.9762	2.4	23.8	2.1	108.2	1.7	10.3	1.5	104.3	1.6	39.6
25	3	2	0.4	41390.96	0.9648	3.5	14.3	2.4	69.3	1.9	12.5	1.7	86.7	1.7	37.0
25	3	2	0.6	43001.19	0.9660	2.8	15.3	1.4	46.1	0.8	11.1	0.6	75.2	0.7	29.7
25	3	2	0.8	48589.33	0.9828	1.0	17.5	0.4	60.6	0.2	9.4	0.1	51.9	0.2	19.2
25	4	2	0.2	33019.26	0.9649	3.6	11.7	2.6	52.0	1.8	11.1	1.5	96.7	1.6	32.8
25	4	2	0.4	36553.23	0.9791	1.9	10.9	0.8	41.0	0.3	10.5	0.1	36.3	0.2	30.4
25	4	2	0.6	38746.77	0.9622	2.7	9.1	1.0	35.4	0.6	10.9	0.5	88.1	0.5	40.8
25	4	2	0.8	48589.33	0.9904	0.5	17.8	0.3	140.6	0.2	9.4	0.2	126.0	0.2	23.1
25	4	3	0.2	33019.26	0.9649	3.6	11.6	2.6	59.7	1.8	9.7	1.5	101.4	1.6	53.4
25	4	3	0.4	36553.23	0.9794	1.8	8.9	0.8	32.1	0.3	9.9	0.1	31.4	0.1	34.2
25	4	3	0.6	38746.77	0.9668	2.2	7.9	0.6	25.8	0.2	11.2	0.1	41.5	0.1	23.3
25	4	3	0.8	48589.33	0.9931	0.2	15.5	0.1	34.0	0.0	6.8	0.0	6.8	0.0	6.8
25	5	2	0.2	30218.49	0.9760	2.4	9.0	1.5	48.3	1.2	11.6	0.8	81.6	0.8	40.1
25	5	2	0.4	32335.62	0.9646	3.1	7.0	1.2	26.5	1.0	10.9	0.8	69.7	0.8	36.1
25	5	2	0.6	36682.65	0.9610	2.5	7.4	1.2	30.5	1.0	11.2	0.9	79.4	1.0	25.9
25	5	2	0.8	48589.33	0.9946	0.2	14.3	0.1	303.5	0.1	7.8	0.1	74.8	0.1	31.0
25	5	3	0.2	30218.49	0.9760	2.4	8.7	1.5	45.7	1.2	9.8	0.8	57.7	0.8	19.3
25	5	3	0.4	32335.62	0.9678	2.7	5.7	0.9	24.9	0.7	10.2	0.4	51.8	0.5	29.7
25	5	3	0.6	36682.65	0.9658	2.0	6.2	0.7	24.7	0.5	10.2	0.4	52.6	0.4	19.5
25	5	3	0.8	48589.33	0.9952	0.2	16.9	0.1	63.1	0.1	7.1	0.1	94.7	0.1	14.8
25	5	4	0.2	30218.49	0.9760	2.4	8.7	1.5	36.8	1.2	9.3	0.8	69.3	0.8	56.4
25	5	4	0.4	32335.62	0.9678	2.7	5.6	0.9	25.0	0.7	10.6	0.3	44.8	0.4	31.5
25	5	4	0.6	36682.65	0.9659	2.0	6.2	0.7	22.7	0.5	9.6	0.5	54.0	0.4	18.8
25	5	4	0.8	48589.33	0.9952	0.2	6.8	0.1	42.5	0.1	6.9	0.1	143.1	0.1	12.6
Avg.						2.1	11.1	1.1	58.3	0.8	9.9	0.6	71.6	0.6	29.4

Table 6: Comparison with the literature for AP50 instances with r-allocation and partial coverage criterion.

										Algorithm (1)					
						State-of-art Lit. Formulation						Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
50	3	2	0.2	45698.26	0.9861	-	-	1.3	7200.0	0.8	294.1	0.7	7200.0	0.7	579.2
50	3	2	0.4	50870.54	0.9923	-	-	0.6	7200.0	0.3	260.6	0.2	7200.0	0.2	355.2
50	3	2	0.6	54337.55	0.9928	-	-	0.4	7200.0	0.1	261.8	0.1	7200.0	0.1	338.4
50	3	2	0.8	56061.85	0.9893	-	-	0.4	7200.0	0.1	286.0	0.1	7200.0	0.1	357.5
50	4	2	0.2	39390.22	0.9889	-	-	0.9	7200.0	0.4	301.6	0.4	7200.0	0.4	560.2
50	4	2	0.4	45575.81	0.9923	-	-	0.6	7200.0	0.3	248.5	0.3	7200.0	0.3	630.8
50	4	2	0.6	48412.31	0.9868	-	-	0.8	7200.0	0.4	285.3	0.4	7200.0	0.4	593.0
50	4	2	0.8	54489.92	0.9893	-	-	0.5	7200.0	0.3	238.9	0.3	7200.0	0.3	570.0
50	4	3	0.2	39390.22	0.9891	-	-	0.9	7200.0	0.4	235.6	0.4	7200.0	0.3	437.1
50	4	3	0.4	45575.81	0.9924	-	-	0.6	7200.0	0.3	247.5	0.2	7200.0	0.3	354.1
50	4	3	0.6	48412.31	0.9880	-	-	0.7	7200.0	0.3	280.0	0.3	7200.0	0.3	393.9
50	4	3	0.8	54489.92	0.9916	-	-	0.3	7200.0	0.1	163.6	0.1	7200.0	0.1	304.6
50	5	2	0.2	32648.59	0.9682	-	-	2.6	7200.0	1.6	338.4	1.5	7200.0	1.5	1323.4
50	5	2	0.4	39975.05	0.9820	-	-	1.4	7200.0	1.0	271.2	1.0	7200.0	1.0	1133.4
50	5	2	0.6	45784.24	0.9856	-	-	0.9	7200.0	0.7	282.3	0.6	7200.0	0.6	963.2
50	5	2	0.8	54088.39	0.9929	-	-	0.3	7200.0	0.2	233.3	0.2	7200.0	0.2	476.7
50	5	3	0.2	32648.59	0.9685	-	-	2.6	7200.0	1.6	315.8	1.5	7200.0	1.5	892.1
50	5	3	0.4	39975.05	0.9839	-	-	1.2	7200.0	0.8	267.5	0.8	7200.0	0.8	621.1
50	5	3	0.6	45784.24	0.9876	-	-	0.7	7200.0	0.5	223.6	0.4	7200.0	0.4	512.2
50	5	3	0.8	54088.39	0.9942	-	-	0.2	7200.0	0.1	164.7	0.1	7200.0	0.1	268.1
50	5	4	0.2	32648.59	0.9685	-	-	2.6	7200.0	1.6	256.8	1.5	7200.0	1.5	993.6
50	5	4	0.4	39975.05	0.9839	-	-	1.2	7200.0	0.8	243.8	0.8	7200.0	0.8	579.3
50	5	4	0.6	45784.24	0.9876	-	-	0.7	7200.0	0.5	211.5	0.5	7200.0	0.4	499.4
50	5	4	0.8	54088.39	0.9944	-	-	0.1	7200.0	0.0	118.9	0.0	7200.0	0.0	214.7
Avg.						-	-	1.0	7200.0	0.5	251.3	0.5	7200.0	0.5	581.3

Table 7: Comparison with the literature for CAB25 instances with multiple allocation and binary coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	2	2	0.2	2136	0.9310	7.0	2.8	3.8	11.2	2.6	3.4	1.3	14.0	1.1	6.2
25	2	2	0.4	2401	0.9605	3.0	2.4	0.1	9.2	0.7	2.9	0.1	7.0	0.0	4.0
25	2	2	0.6	2557	0.9396	3.4	1.9	0.7	3.7	0.0	2.7	6.4	5.9	6.4	3.6
25	2	2	0.8	2713	0.8992	2.6	1.4	1.2	3.5	1.5	2.3	0.9	8.2	1.0	4.7
25	3	3	0.2	1913	0.9658	3.5	1.9	2.3	9.6	2.1	3.4	1.5	16.9	1.3	6.4
25	3	3	0.4	2099	0.9624	3.6	1.8	0.8	8.2	0.7	3.0	0.2	15.4	0.1	4.7
25	3	3	0.6	2336	0.9317	4.7	1.4	1.5	5.2	1.9	2.4	0.8	9.7	0.8	4.7
25	3	3	0.8	2552	0.9008	1.5	1.4	0.0	5.8	0.2	2.3	0.0	5.9	0.0	3.6
25	4	4	0.2	1617	0.9571	4.5	1.8	2.8	8.0	2.7	2.6	2.1	15.8	2.4	8.1
25	4	4	0.4	1881	0.9501	4.9	1.5	1.4	8.2	2.1	2.8	1.4	16.7	1.0	5.1
25	4	4	0.6	2184	0.9363	3.5	1.3	0.5	5.8	0.5	2.5	0.2	10.2	0.1	4.1
25	4	4	0.8	2457	0.8961	1.5	1.2	0.4	3.5	0.6	2.5	0.4	9.0	0.4	4.5
25	5	5	0.2	1346	0.9270	7.7	1.4	3.1	8.9	3.6	2.5	2.6	17.2	2.4	5.4
25	5	5	0.4	1597	0.9184	6.2	1.2	1.7	5.1	1.2	2.4	0.5	9.4	0.6	4.2
25	5	5	0.6	2002	0.9019	3.9	1.2	0.8	5.5	1.3	2.3	0.6	14.7	0.9	5.4
25	5	5	0.8	2307	0.8905	0.9	1.1	0.1	3.0	0.4	2.0	0.1	7.7	0.0	3.7
Avg.						3.9	1.6	1.3	6.5	1.4	2.6	1.2	11.5	1.2	4.9

Table 8: Comparison with the literature for AP25 instances with multiple allocation and binary coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	2	2	0.2	45813.71	0.9441	5.7	2.8	3.4	11.7	2.8	3.5	0.6	41.9	1.5	7.7
25	2	2	0.4	47336.19	0.9392	5.4	1.9	2.5	12.9	2.5	2.8	0.1	33.0	1.4	6.9
25	2	2	0.6	49123.94	0.9476	3.4	1.7	0.7	7.7	0.9	2.8	3.0	15.2	0.4	5.0
25	2	2	0.8	53112.15	0.9681	0.8	1.6	0.0	3.3	0.0	2.3	0.0	21.1	3.3	3.3
25	3	3	0.2	40780.89	0.9623	3.9	2.2	3.1	12.5	2.6	3.4	1.4	45.2	1.5	9.0
25	3	3	0.4	41390.96	0.9474	5.1	1.7	2.9	9.9	2.6	2.7	1.3	54.5	1.6	5.9
25	3	3	0.6	43001.19	0.9401	4.3	1.5	1.7	7.1	1.7	2.5	0.2	40.3	1.0	4.8
25	3	3	0.8	48589.33	0.9733	0.7	1.4	0.0	4.4	0.1	2.3	1.4	11.7	0.1	3.3
25	4	4	0.2	33019.26	0.9314	7.3	1.7	4.8	10.2	4.0	2.8	1.4	44.7	3.1	8.0
25	4	4	0.4	36553.23	0.9571	3.7	1.4	1.1	8.3	1.1	2.5	0.1	22.6	0.4	4.3
25	4	4	0.6	38746.77	0.9378	3.3	1.2	0.6	6.5	1.1	2.2	0.0	27.4	0.6	4.3
25	4	4	0.8	48589.33	0.9859	0.4	1.3	0.1	3.2	0.1	2.5	0.7	12.6	0.0	3.6
25	5	5	0.2	30218.49	0.9534	4.9	1.4	2.5	7.8	2.2	2.6	0.7	33.7	1.1	5.0
25	5	5	0.4	32335.62	0.9392	4.9	1.2	1.2	6.2	1.7	2.3	0.3	30.9	1.0	4.6
25	5	5	0.6	36682.65	0.9367	3.1	1.2	1.2	4.5	1.5	2.2	0.4	36.5	1.1	5.2
25	5	5	0.8	48589.33	0.9901	0.4	1.1	0.2	3.0	0.2	2.0	0.1	40.4	0.1	3.9
Avg.						3.6	1.6	1.6	7.5	1.6	2.6	0.7	32.0	1.1	5.3

Table 9: Comparison with the literature for AP50 instances with multiple allocation and binary coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
50	2	2	0.2	56118.4	0.9832	1.7	368.1	1.5	1284.9	0.9	49.9	0.6	1551.2	0.6	108.9
50	2	2	0.4	57861.04	0.9801	1.7	281.7	1.4	765.7	0.8	36.8	0.6	945.7	0.5	53.9
50	2	2	0.6	59549.3	0.9750	1.8	153.3	1.6	516.8	1.0	33.0	0.8	1085.7	0.8	85.4
50	2	2	0.8	61072.99	0.9788	0.9	68.9	0.6	243.8	0.3	21.3	0.2	483.1	0.2	30.0
50	3	3	0.2	45698.26	0.9708	3.0	205.5	2.7	2016.5	2.0	39.9	1.7	2549.6	1.7	137.8
50	3	3	0.4	50870.54	0.9842	1.5	182.6	1.3	802.5	0.7	32.5	0.5	1143.4	0.5	48.0
50	3	3	0.6	54337.55	0.9840	1.1	85.2	0.9	420.6	0.5	26.4	0.4	1011.5	0.4	42.9
50	3	3	0.8	56061.85	0.9804	0.7	67.1	0.5	229.1	0.3	19.0	0.2	516.1	0.2	28.0
50	4	4	0.2	39390.22	0.9804	1.9	143.9	1.5	1559.2	0.7	33.0	0.5	1213.7	0.6	52.5
50	4	4	0.4	45575.81	0.9855	1.3	111.4	1.0	1017.7	0.6	32.9	0.4	1847.1	0.4	48.3
50	4	4	0.6	48412.31	0.9779	1.4	71.7	1.1	875.3	0.8	24.2	0.7	1407.0	0.7	51.5
50	4	4	0.8	54489.92	0.9831	0.6	66.2	0.5	328.6	0.3	16.3	0.3	1131.4	0.3	29.8
50	5	5	0.2	32648.59	0.9483	5.1	75.1	4.1	4931.5	2.9	24.8	2.7	1652.5	2.6	113.9
50	5	5	0.4	39975.05	0.9707	2.6	66.1	2.1	1336.0	1.6	23.5	1.4	1895.9	1.4	76.2
50	5	5	0.6	45784.24	0.9771	1.5	47.2	1.2	876.8	1.0	17.8	0.9	1612.9	0.9	105.1
50	5	5	0.8	54088.39	0.9875	0.4	39.6	0.3	287.8	0.2	16.0	0.2	1604.0	0.2	43.8
Avg.						1.7	127.1	1.4	1093.3	0.9	28.0	0.8	1353.2	0.8	66.0

Table 10: Comparison with the literature for CAB25 instances with multiple allocation and partial coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	2	2	0.2	2136	0.9701	3.0	9.4	0.8	20.5	0.6	8.1	0.1	16.5	0.3	10.1
25	2	2	0.4	2401	0.9771	2.1	7.3	0.0	17.1	0.2	8.6	0.0	17.8	0.0	11.5
25	2	2	0.6	2557	0.9620	3.0	5.8	0.0	13.6	0.0	8.6	3.9	15.5	0.0	11.3
25	2	2	0.8	2713	0.9417	3.1	4.6	0.2	11.9	0.5	6.2	0.2	25.3	0.2	11.0
25	3	3	0.2	1913	0.9826	1.8	5.8	1.4	18.9	1.0	9.4	0.8	39.3	0.6	29.9
25	3	3	0.4	2099	0.9723	2.8	6.1	1.0	20.2	0.8	9.6	0.6	42.1	0.5	14.1
25	3	3	0.6	2336	0.9575	3.6	4.6	0.8	13.3	0.6	8.1	0.4	29.8	0.4	15.5
25	3	3	0.8	2552	0.9366	2.4	3.9	0.2	10.9	0.2	6.9	0.1	20.4	0.1	10.7
25	4	4	0.2	1617	0.9714	2.9	4.4	2.0	15.9	1.9	8.1	1.6	49.5	1.7	26.3
25	4	4	0.4	1881	0.9690	3.1	4.2	1.1	19.1	1.0	9.1	0.7	42.5	0.6	14.8
25	4	4	0.6	2184	0.9612	2.9	3.4	0.3	11.2	0.1	9.9	0.0	16.5	0.0	13.0
25	4	4	0.8	2457	0.9358	1.8	3.1	0.0	6.2	0.0	6.5	0.0	13.2	0.0	8.4
25	5	5	0.2	1346	0.9491	5.3	3.3	2.6	18.1	3.1	9.7	2.6	73.6	2.5	39.8
25	5	5	0.4	1597	0.9512	4.1	2.9	0.8	8.2	0.4	8.5	0.2	23.4	0.1	21.0
25	5	5	0.6	2002	0.9461	2.9	2.7	0.3	9.8	0.2	9.5	0.0	19.8	0.0	12.7
25	5	5	0.8	2307	0.9151	1.2	2.5	0.1	8.0	0.2	6.6	0.1	20.4	0.2	11.4
Avg.						2.9	4.6	0.7	13.9	0.7	8.3	0.7	29.1	0.5	16.4

Table 11: Comparison with the literature for AP25 instances with multiple allocation and partial coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	2	2	0.2	45813.71	0.9698	3.0	10.2	1.6	25.2	0.9	14.3	0.6	41.9	0.3	18.9
25	2	2	0.4	47336.19	0.9697	2.8	8.9	1.2	20.3	0.4	11.8	0.1	33.0	0.0	14.8
25	2	2	0.6	49123.94	0.9709	2.4	7.0	0.3	14.7	0.1	8.6	3.0	15.2	0.0	10.3
25	2	2	0.8	53112.15	0.9804	1.2	5.4	0.2	13.6	0.1	7.3	0.0	21.1	0.1	10.5
25	3	3	0.2	40780.89	0.9764	2.4	7.7	2.0	33.2	1.7	9.4	1.4	45.2	1.3	22.4
25	3	3	0.4	41390.96	0.9656	3.4	5.8	2.2	19.1	1.8	11.9	1.3	54.5	1.3	24.4
25	3	3	0.6	43001.19	0.9691	2.5	4.3	0.8	13.3	0.5	10.1	0.2	40.3	0.2	14.7
25	3	3	0.8	48589.33	0.9865	0.6	4.4	0.0	8.5	0.0	5.6	1.4	11.7	0.0	7.5
25	4	4	0.2	33019.26	0.9649	3.6	5.1	2.4	18.9	1.8	9.3	1.4	44.7	1.4	25.9
25	4	4	0.4	36553.23	0.9794	1.8	4.3	0.7	12.1	0.3	9.7	0.1	22.6	0.0	13.0
25	4	4	0.6	38746.77	0.9668	2.2	3.4	0.4	10.5	0.2	10.6	0.0	27.4	0.1	14.4
25	4	4	0.8	48589.33	0.9931	0.2	3.9	0.0	8.5	0.0	6.5	0.7	12.6	0.0	8.2
25	5	5	0.2	30218.49	0.9760	2.4	4.0	1.5	21.7	1.2	9.0	0.7	33.7	0.7	21.3
25	5	5	0.4	32335.62	0.9678	2.7	3.2	0.7	12.0	0.7	10.1	0.3	30.9	0.4	19.5
25	5	5	0.6	36682.65	0.9659	2.0	2.8	0.6	9.6	0.5	9.0	0.4	36.5	0.4	17.7
25	5	5	0.8	48589.33	0.9952	0.2	3.6	0.1	8.1	0.1	5.4	0.1	40.4	0.1	11.3
Avg.						2.1	5.3	0.9	15.6	0.6	9.3	0.7	32.0	0.4	15.9

Table 12: Comparison with the literature for AP50 instances with multiple allocation and partial coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
50	2	2	0.2	56118.4	0.9919	0.8	1222.9	0.7	7200.0	0.3	241.2	0.2	6336.3	0.1	309.7
50	2	2	0.4	57861.04	0.9903	0.9	1230.3	0.7	7200.0	0.2	279.3	0.1	4088.8	0.1	335.9
50	2	2	0.6	59549.3	0.9883	1.0	800.0	0.7	3898.5	0.2	247.7	0.2	4077.4	0.2	353.6
50	2	2	0.8	61072.99	0.9904	0.5	601.3	0.3	2302.3	0.0	151.3	0.0	822.8	0.0	171.7
50	3	3	0.2	45698.26	0.9861	1.4	889.5	1.4	7200.0	0.8	259.8	0.7	7200.0	0.6	663.8
50	3	3	0.4	50870.54	0.9925	0.7	786.4	0.8	7200.0	0.3	236.1	0.2	6430.5	0.2	319.0
50	3	3	0.6	54337.55	0.9930	0.6	558.6	0.4	3955.0	0.1	220.6	0.1	3238.3	0.1	381.4
50	3	3	0.8	56061.85	0.9904	0.5	424.7	0.3	2190.0	0.1	172.7	0.1	2280.7	0.1	237.9
50	4	4	0.2	39390.22	0.9891	1.1	835.5	0.9	7200.0	0.4	230.3	0.3	4793.9	0.3	333.3
50	4	4	0.4	45575.81	0.9924	0.7	767.9	0.9	7200.0	0.3	203.2	0.2	4965.8	0.2	271.0
50	4	4	0.6	48412.31	0.9880	0.9	472.3	0.7	7200.0	0.3	224.5	0.3	5851.2	0.3	322.6
50	4	4	0.8	54489.92	0.9916	0.4	391.5	0.3	4401.3	0.1	139.4	0.1	4461.0	0.1	219.2
50	5	5	0.2	32648.59	0.9685	3.2	396.6	2.8	7200.0	1.6	273.6	1.4	4741.1	1.4	1226.3
50	5	5	0.4	39975.05	0.9839	1.5	355.1	1.4	7200.0	0.8	236.3	0.8	6028.0	0.8	538.1
50	5	5	0.6	45784.24	0.9876	0.9	391.4	0.9	7200.0	0.5	200.4	0.4	7200.0	0.4	507.4
50	5	5	0.8	54088.39	0.9944	0.2	316.1	0.1	7200.0	0.0	101.9	0.0	3220.8	0.0	139.1
Avg.						1.0	652.5	0.8	5996.7	0.4	213.6	0.3	4733.5	0.3	395.6

Table 13: Comparison with the literature for CAB instances with single-allocation and binary coverage criterion.

										Algorithm (1)					
						State-of-art Lit. Formulation						Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	2	1	0.2	2136	0.9266	7.8	1.0	6.2	10.1	3.1	3.8	1.9	16.6	2.3	9.5
25	2	1	0.4	2401	0.9401	6.0	1.0	4.1	7.3	1.9	3.1	0.9	13.7	1.2	6.6
25	2	1	0.6	2557	0.9001	10.3	1.0	5.3	7.3	1.3	3.2	0.6	14.3	0.9	7.0
25	2	1	0.8	2713	0.8779	9.5	0.9	2.6	5.2	0.0	2.6	0.0	2.6	0.0	2.6
25	3	1	0.2	1913	0.9597	4.2	0.9	3.9	15.1	2.7	5.5	2.0	16.6	2.3	12.3
25	3	1	0.4	2099	0.9540	4.5	1.0	2.6	7.2	1.1	3.3	0.5	9.3	0.9	5.9
25	3	1	0.6	2336	0.9191	8.1	0.9	3.3	6.5	0.6	3.1	0.0	7.5	0.4	5.3
25	3	1	0.8	2552	0.8735	5.4	0.9	2.3	4.5	0.8	2.9	0.3	8.2	0.7	7.0
25	4	1	0.2	1617	0.9567	4.5	0.9	3.7	7.8	2.7	4.7	2.1	18.4	2.4	11.4
25	4	1	0.4	1881	0.9439	5.6	1.0	3.6	5.9	2.1	3.6	1.4	11.3	1.6	6.0
25	4	1	0.6	2184	0.9152	8.5	0.9	2.9	6.1	1.2	3.2	0.7	8.1	0.9	5.6
25	4	1	0.8	2457	0.8730	4.7	0.9	2.1	5.2	1.4	2.7	0.8	8.0	1.0	7.9
25	5	1	0.2	1346	0.9238	8.0	1.0	5.6	14.4	3.9	3.9	2.2	13.8	2.9	11.2
25	5	1	0.4	1597	0.8920	11.3	1.1	5.6	6.5	3.1	3.3	2.4	11.9	2.6	8.1
25	5	1	0.6	2002	0.8838	8.5	0.9	2.6	5.0	2.0	2.8	1.2	9.5	1.3	8.4
25	5	1	0.8	2307	0.8623	4.8	0.9	1.5	3.8	1.1	2.7	0.6	7.9	0.8	5.7
Avg.						7.0	1.0	3.6	7.4	1.8	3.4	1.1	11.1	1.4	7.5

Table 14: Comparison with the literature for AP25 instances with single-allocation and binary coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	2	1	0.2	45813.71	0.9389	6.5	0.9	5.3	11.1	3.3	5.4	2.4	19.1	3.1	18.5
25	2	1	0.4	47336.19	0.9307	7.3	1.0	5.8	10.9	3.4	3.9	2.3	18.6	3.0	12.3
25	2	1	0.6	49123.94	0.9336	6.8	1.0	4.8	10.3	2.0	3.6	1.5	11.7	1.7	8.3
25	2	1	0.8	53112.15	0.9506	4.5	1.0	2.2	6.5	0.7	2.8	0.4	8.2	0.5	5.3
25	3	1	0.2	40780.89	0.9553	4.7	0.9	3.9	15.7	3.3	7.3	2.1	19.1	2.8	22.1
25	3	1	0.4	41390.96	0.9361	6.7	0.9	5.7	18.0	3.5	4.9	2.5	18.4	3.2	11.1
25	3	1	0.6	43001.19	0.9188	8.4	0.9	6.0	13.4	2.8	3.1	2.1	11.6	2.4	13.6
25	3	1	0.8	48589.33	0.9462	5.0	1.0	2.6	7.2	1.0	2.6	0.4	8.0	0.7	5.6
25	4	1	0.2	33019.26	0.9305	7.5	0.9	6.0	35.4	4.1	4.5	2.4	18.6	3.5	15.1
25	4	1	0.4	36553.23	0.9279	7.7	0.9	6.5	26.0	3.6	3.2	2.7	13.9	3.2	13.7
25	4	1	0.6	38746.77	0.8997	10.4	1.0	7.3	23.2	3.9	2.9	3.2	12.6	3.5	12.2
25	4	1	0.8	48589.33	0.9590	3.8	1.0	2.2	10.0	1.2	2.5	0.9	9.4	1.1	6.1
25	5	1	0.2	30218.49	0.9523	5.0	0.9	3.9	21.0	2.2	4.4	0.9	10.3	1.5	10.3
25	5	1	0.4	32335.62	0.9145	9.1	0.9	6.9	54.3	3.7	2.8	2.4	15.6	3.3	13.7
25	5	1	0.6	36682.65	0.9033	9.9	1.0	6.6	28.2	3.7	2.7	3.1	15.4	3.4	10.0
25	5	1	0.8	48589.33	0.9770	2.0	0.9	0.9	4.8	0.3	2.6	0.1	6.0	0.1	5.5
Avg.						6.6	0.9	4.8	18.5	2.7	3.7	1.8	13.5	2.3	11.5

Table 15: Comparison with the literature for AP50 instances with single-allocation and binary coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
50	2	1	0.2	56118.4	0.9827	1.8	274.1	1.6	1095.3	0.9	391.4	0.8	1020.0	0.9	618.1
50	2	1	0.4	57861.04	0.9771	2.3	176.7	2.2	1582.0	1.1	131.9	1.0	876.4	1.1	383.7
50	2	1	0.6	59549.3	0.9706	3.0	26.1	2.7	1014.4	1.3	60.5	1.2	458.6	1.3	174.1
50	2	1	0.8	61072.99	0.9679	3.1	9.1	2.3	695.2	0.9	45.3	0.9	361.1	0.9	93.2
50	3	1	0.2	45698.26	0.9698	3.4	86.6	3.2	7200.0	2.1	310.9	1.8	1243.2	2.0	1679.4
50	3	1	0.4	50870.54	0.9773	2.3	133.5	2.2	7200.0	1.3	127.8	1.2	635.6	1.2	237.9
50	3	1	0.6	54337.55	0.9751	2.5	48.3	2.2	4843.9	1.1	56.1	1.0	479.1	1.0	185.4
50	3	1	0.8	56061.85	0.9661	3.2	6.1	2.2	3005.6	0.9	35.6	0.8	208.5	0.8	90.9
50	4	1	0.2	39390.22	0.9775	2.7	106.4	2.6	7200.0	1.0	216.5	0.8	467.4	0.9	271.2
50	4	1	0.4	45575.81	0.9786	2.4	28.9	2.3	7200.0	1.2	87.3	1.0	540.0	1.1	148.6
50	4	1	0.6	48412.31	0.9618	3.9	6.1	3.4	7200.0	1.9	40.8	1.8	627.7	1.9	266.7
50	4	1	0.8	54489.92	0.9707	2.6	6.9	1.8	2397.2	0.8	38.0	0.7	229.9	0.7	108.1
50	5	1	0.2	32648.59	0.9435	7.6	41.8	7.3	7200.0	3.4	112.0	3.2	1309.2	3.4	1239.1
50	5	1	0.4	39975.05	0.9596	4.6	12.6	4.3	7200.0	2.5	56.0	2.3	941.0	2.5	415.5
50	5	1	0.6	45784.24	0.9635	3.6	6.8	3.0	7200.0	1.8	44.2	1.7	645.8	1.7	380.4
50	5	1	0.8	54088.39	0.9767	2.0	5.9	1.4	3850.3	0.7	34.9	0.6	245.9	0.7	116.3
Avg.						3.2	61.0	2.8	4755.2	1.4	111.8	1.3	643.1	1.4	400.5

Table 16: Comparison with the literature for CAB instances with single-allocation and partial coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	2	1	0.2	2136	0.9619	4.0	1.1	4.0	11.8	1.2	11.9	0.6	33.6	0.8	39.2
25	2	1	0.4	2401	0.9654	3.6	1.0	3.5	11.4	0.9	12.0	0.5	31.0	0.7	19.6
25	2	1	0.6	2557	0.9347	6.7	1.1	6.6	22.5	1.1	12.9	0.8	36.2	0.9	21.8
25	2	1	0.8	2713	0.9183	7.6	1.0	7.5	29.3	0.1	10.9	0.0	14.4	0.0	13.0
25	3	1	0.2	1913	0.9776	2.3	0.9	2.3	26.8	1.5	12.8	1.2	87.1	1.3	53.6
25	3	1	0.4	2099	0.9672	3.4	1.0	3.3	44.8	1.0	13.4	0.8	31.6	0.9	23.8
25	3	1	0.6	2336	0.9402	6.2	1.1	6.0	52.5	1.2	12.0	0.9	35.0	1.0	36.0
25	3	1	0.8	2552	0.9086	6.8	1.0	6.7	62.6	1.0	12.7	0.7	34.8	0.8	27.3
25	4	1	0.2	1617	0.9707	3.0	0.9	3.0	36.7	1.9	14.2	1.7	64.0	1.8	54.1
25	4	1	0.4	1881	0.9649	3.6	1.0	3.5	65.1	1.0	11.2	0.7	29.2	0.8	18.2
25	4	1	0.6	2184	0.9462	5.4	1.0	5.3	151.2	0.4	12.4	0.3	27.3	0.3	21.1
25	4	1	0.8	2457	0.9087	6.4	1.0	6.2	351.2	1.1	13.9	0.7	35.9	0.9	25.6
25	5	1	0.2	1346	0.9468	5.6	1.0	5.6	294.8	3.2	11.2	2.5	68.1	2.6	73.8
25	5	1	0.4	1597	0.9340	6.8	1.0	6.5	98.7	1.3	12.4	0.9	38.7	1.0	25.8
25	5	1	0.6	2002	0.9300	6.2	0.9	5.9	298.8	0.5	12.4	0.3	35.3	0.4	24.2
25	5	1	0.8	2307	0.8929	4.9	1.0	4.8	190.1	0.8	11.5	0.6	34.6	0.7	29.2
Avg.						5.2	1.0	5.0	109.3	1.1	12.4	0.8	39.8	0.9	31.7

Table 17: Comparison with the literature for AP25 instances with single-allocation and partial coverage criterion.

										Algorithm (1)					
						State-of-art Lit. Formulation						Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
25	2	1	0.2	45813.71	0.9640	3.7	0.9	3.6	29.0	1.5	16.7	1.0	49.3	1.3	41.1
25	2	1	0.4	47336.19	0.9591	4.3	1.1	4.1	29.1	1.3	15.9	0.9	47.6	1.1	36.7
25	2	1	0.6	49123.94	0.9597	4.1	1.0	4.0	25.7	0.6	18.1	0.4	40.1	0.5	34.5
25	2	1	0.8	53112.15	0.9696	2.9	0.9	2.8	21.9	0.2	13.9	0.1	27.6	0.2	20.6
25	3	1	0.2	40780.89	0.9723	2.8	0.9	2.8	186.9	2.1	10.9	1.6	60.9	1.9	40.2
25	3	1	0.4	41390.96	0.9580	4.4	1.2	4.2	258.3	2.3	15.0	1.9	73.0	2.1	39.1
25	3	1	0.6	43001.19	0.9452	5.7	0.9	5.4	344.4	1.9	15.5	1.6	66.9	1.7	41.4
25	3	1	0.8	48589.33	0.9646	3.5	0.9	3.3	208.2	0.6	13.9	0.5	37.7	0.5	23.3
25	4	1	0.2	33019.26	0.9602	4.1	0.8	4.0	516.0	2.3	12.7	1.6	70.5	2.0	53.5
25	4	1	0.4	36553.23	0.9604	4.1	0.9	3.9	250.2	1.6	16.0	1.2	50.0	1.4	47.4
25	4	1	0.6	38746.77	0.9327	7.0	0.9	6.7	631.8	2.3	15.7	2.0	83.6	2.1	67.4
25	4	1	0.8	48589.33	0.9839	1.5	0.9	1.4	89.9	0.0	8.3	0.0	8.3	0.0	8.3
25	5	1	0.2	30218.49	0.9742	2.6	0.8	2.5	154.4	1.3	12.8	0.6	32.4	1.0	24.2
25	5	1	0.4	32335.62	0.9465	5.6	0.9	5.3	592.1	2.2	15.3	1.5	72.9	1.9	71.5
25	5	1	0.6	36682.65	0.9357	6.7	0.9	6.2	1228.5	2.3	13.7	2.0	111.4	2.2	102.6
25	5	1	0.8	48589.33	0.9888	1.0	0.9	1.0	55.8	0.1	11.8	0.0	21.9	0.1	20.8
Avg.						4.0	0.9	3.8	288.9	1.4	14.1	1.1	53.4	1.2	42.0

Table 18: Comparison with the literature for AP50 instances with single-allocation and partial coverage criterion.

						State-of-art Lit. Formulation				Algorithm (1)					
												Adapted Lit. Formulation		Proposed Formulation (5)	
n	p	r	α	β	opt	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0	$T_0(s)$	Gap_0^{cplex}	$T(s)$	Gap_0^{cplex}	$T(s)$
50	2	1	0.2	56118.4	0.9914	0.9	355.3	0.9	5060.8	0.3	463.4	0.2	851.8	0.3	567.7
50	2	1	0.4	57861.04	0.9888	1.1	384.0	1.1	7200.0	0.3	518.4	0.2	913.0	0.3	752.2
50	2	1	0.6	59549.3	0.9833	1.8	127.8	1.8	7200.0	0.5	557.5	0.5	1727.6	0.5	958.2
50	2	1	0.8	61072.99	0.9820	1.8	76.0	1.8	7200.0	0.4	450.2	0.4	1382.3	0.4	647.3
50	3	1	0.2	45698.26	0.9835	2.0	228.9	1.9	7200.0	1.0	528.4	0.9	2386.5	1.0	953.3
50	3	1	0.4	50870.54	0.9898	1.8	375.5	1.7	7200.0	0.4	480.4	0.4	2296.5	0.4	731.0
50	3	1	0.6	54337.55	0.9877	1.7	579.3	1.7	7200.0	0.3	516.6	0.3	1178.7	0.3	679.7
50	3	1	0.8	56061.85	0.9818	2.3	6.6	2.2	7200.0	0.2	498.8	0.2	879.9	0.2	633.5
50	4	1	0.2	39390.22	0.9877	2.0	224.0	1.9	7200.0	0.5	471.5	0.4	1518.6	0.5	1147.7
50	4	1	0.4	45575.81	0.9888	2.1	263.5	2.0	7200.0	0.5	460.4	0.5	1703.3	0.5	1151.2
50	4	1	0.6	48412.31	0.9791	2.6	61.0	2.5	7200.0	0.8	455.2	0.8	2921.9	0.8	1080.1
50	4	1	0.8	54489.92	0.9842	1.7	40.5	1.7	7200.0	0.2	421.4	0.2	934.4	0.2	594.1
50	5	1	0.2	32648.59	0.9644	5.3	142.6	5.3	7200.0	2.0	490.5	1.8	3646.2	1.9	2583.2
50	5	1	0.4	39975.05	0.9764	2.7	159.4	2.7	7200.0	1.4	477.4	1.3	3817.7	1.4	1567.3
50	5	1	0.6	45784.24	0.9779	2.6	77.8	2.5	7200.0	1.0	407.1	0.9	2624.9	1.0	1502.0
50	5	1	0.8	54088.39	0.9883	1.5	60.4	1.5	7200.0	0.2	379.3	0.2	1035.4	0.2	692.5
Avg.						2.1	197.7	2.1	7066.3	0.6	473.5	0.6	1863.7	0.6	1015.1